

$$1) \frac{dy}{dx} + \frac{x}{y} = 0$$

$$y(0) = 1$$

$$c=1 \quad 2c=2 \quad (\sqrt{2})^2$$

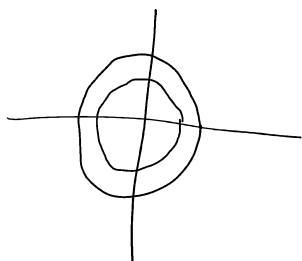
$$y dy + x dx = 0$$

$$\int y dy + \int x dx = 0$$

$$\frac{y^2}{2} + \frac{x^2}{2} = C \Rightarrow$$

$$x^2 + y^2 = 2C = C^2 - \textcircled{1}$$

$$0 + 1 = C^2 \Rightarrow C = 1$$



① $\rightarrow$ is a general sol.

② $\rightarrow$ is a one parameter family of sol.

① $\Rightarrow x^2 + y^2 = 1 \rightarrow$ particular sol.

$$2) (y')^2 - 4y = 0$$

$$y(0) = 0 \quad \text{Find G.S. \& P.S.}$$

$$\frac{dy}{dx} = \pm 2\sqrt{y}$$

$$\Rightarrow y^{-1/2} dy = \pm 2 dx$$

$$\int y^{-1/2} dy = \pm 2 \int dx$$

$$2y^{1/2} = \pm 2x + C$$

$$y^{1/2} = \pm x + C$$

$$y = (\pm x + C)^2 \quad \text{G.S.}$$

$$y(0) = 0$$

at $x=0, y=0$

$$\left(\begin{array}{l} 0 = (\pm 0 + C)^2 \Rightarrow C = 0 \\ y = x^2 \quad \text{P.S.} \end{array} \right.$$

First order ODE & Initial value problem (I.V.P.).

$$\frac{dy}{dx} = f(x, y)$$

with $y(a) = b$ I.V.P. ✓

$$\boxed{y'(a) = b \quad \text{B.V.P.} \times}$$

$$1) \boxed{\frac{dy}{dx} = 2x \quad y(0) = 0}$$

I.V.P.

$$y = x^2 + C$$

$$\text{At } x=0, y=0 \Rightarrow C=0$$

$$y = x^2$$

unique sol.

$$2) \quad x \frac{dy}{dx} = y-1 \quad y(0) = 1$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x} \Rightarrow \log(y-1) = \log x + \log C$$

$$\log(y-1) = \log xC$$

$$\text{At } x=0, y=1$$

$$y-1 = xC$$

$$1-1 = 0 \cdot C \Rightarrow \underline{0 = 0 \cdot C}$$

In finitely many sol.

$$3) \quad \left| \frac{dy}{dx} \right| + |y| = 0$$

$$y(0) = 1 \quad \text{No. sol.}$$

$$\frac{dy}{dx} + y = 0$$

D.E. of first order & first degree $\frac{dy}{dx} = f(x, y)$

Method I: Variable separable: If a D.E. can be written in the form

$$f_1(x) dx + f_2(y) dy = 0$$

$$\int f_1(x) dx + \int f_2(y) dy = C$$

$$\phi_1(x) + \phi_2(y) = C \rightarrow \text{G.S.}$$

$$1) \quad \frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^{-y} (e^x + x^2)$$

→ 1 more step

$$\int e^y dy = \int (e^x + x^2) dx$$

$$e^y = e^x + \frac{x^3}{3} + C$$

$$2) \quad 3e^x \tan y dx + (1-e^x) \sec^2 y dy = 0$$

$$3e^x \tan y dx = (e^x - 1) \sec^2 y dy$$

$$\int 3e^x \tan y \, dx = (e^x - 1) \sec^2 y \, dy$$

$$\int \frac{3e^x}{e^x - 1} \, dx = \int \frac{\sec^2 y}{\tan y} \, dy$$

$$e^x - 1 = t$$

$$e^x \, dx = dt$$

$$\tan y = w$$

$$\sec^2 y \, dy = dw$$

$$3 \int \frac{t}{t} \, dt = \int \frac{1}{w} \, dw$$

$$3 \log (e^x - 1) = \log \tan y + \log c$$

$$\log (e^x - 1)^3 = \log (c \tan y)$$

$$(e^x - 1)^3 = c \tan y \quad \text{Ans.}$$

Method II: Eqⁿ. reducible to variable separable.

$$\frac{dy}{dx} = f(ax + by + c)$$

$$v = ax + by + c$$

$$\frac{dv}{dx} = a + b \frac{dy}{dx}$$

$$\frac{1}{b} \left[\frac{dv}{dx} - a \right] = f(v)$$

$$\frac{dv}{dx} = a + b f(v)$$

$$\Rightarrow \frac{dv}{a + b f(v)} = dx$$

$$\Rightarrow F(v) = x + C$$

$$1) \frac{dy}{dx} = (4x + y + 1)^2$$

$$v = 4x + y + 1$$

$$\frac{dv}{dx} = 4 + \frac{dy}{dx}$$

$$\frac{dv}{dx} - 4 = v^2$$

$$\frac{dv}{dx} = v^2 + 4 \Rightarrow \int \frac{dv}{v^2 + 4} = \int dx$$

$$\Rightarrow \frac{1}{2} \tan^{-1} \frac{v}{2} = x + C$$

$$\Rightarrow \tan^{-1} \frac{v}{2} = 2x + C_1$$

$$v = \tan (2x + C_1)$$

$$\Rightarrow \frac{v}{a} = \tan(2x + C)$$

$$y + x + 1 = 2 \tan(2x + C) \quad \underline{\text{Ans.}}$$

$$2) (x+y)^2 \frac{dy}{dx} = a^2$$

$$v^2 \left[\frac{dv}{dx} - 1 \right] = a^2$$

$$\frac{dv}{dx} = \frac{a^2}{v^2} + 1 = \frac{a^2 + v^2}{v^2}$$

$$\frac{v^2}{v^2 + a^2} dv = dx$$

$$\frac{v^2 + a^2 - a^2}{v^2 + a^2} dv = dx \Rightarrow \int \left(1 - \frac{a^2}{v^2 + a^2} \right) dv = \int dx$$

$$v - a^2 \cdot \frac{1}{a} \tan^{-1} \frac{v}{a} = x + C$$

$$v - a \tan^{-1} \frac{v}{a} = x + C$$

$$x + y - a \tan^{-1} \frac{x+y}{a} = x + C$$

$$a \tan^{-1} \frac{x+y}{a} = y - C$$

$$\frac{x+y}{a} = \tan \left(\frac{y-C}{a} \right)$$

Ans.